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The asynchronous iterations applied to the modified porous meduim equation

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***Abstract*: The purpose of this work is to establish the existence and uniqueness of the solution of the porous medium equation where the solution is subject to certain constraints, for the boundary conditions of Dirichlet and resolve it by different numerical methods.**

After a appropriate change of variables, we consider an implicit time discretization scheme that leads to solve at each time a sequence of nonlinear stationary problems. For each stationary problem, we use a spatial discretization that leads to each time to solve a multivalued large nonlinear algebraic system. Finally, we applied subdomain asynchronous parallel methods without overlapping and we established the convergence by a contraction technique.

***Keywords*: Optimisation problem; Parallel solution; Hydrodynamic limit; Relaxation methods**

INTRODUCTION

One of the main motivations of the physical study of the hydrodynamic limit comes from a branch of statistical mechanics. The goal is to characterize the macroscopic equation governing the evolution of a fluid or gas from a microscopic random dynamic. At the microscopic level the evolution of the particles is modeled on a microscopic volume according to the initial profile, after renormalization in space and time, at the time ***t*** the system is described by the density of particles which is the solution of the partial differential equation of the parabolic type under the scaling change, it called hydrodynamic equation, that describe the spatial and temporal evolution of the macroscopic variables of a fluid or gas evolving in a volume from a microscopic dynamics at random due to the large number of particles. The purpose of this work is to treat numerically the hydrodynamic equations by numerical methods with homogeneous boundary conditions of Dirichlet

1-Problem presentation

First, We consider the boundary value problem (1.1) equipped with homogeneous Dirichlet boundary condition then we have to find the solution *u(t,x)* , the density function, which satisfies the following boundary value problem :

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} - \Delta\left(\left(\dfrac{u(t,x)}{u(t,x)+\theta}\right)^m\right) = 0 \;; t \in [0,\mathrm{T}], x \in [0,1[^d \\ u(0,x) = u_0(x) = \phi(x) \\ u(t,x) = 0 \qquad si\ \ u \in \partial\Omega \end{cases} \tag{1.1}$$

where $0 = \delta_0 \le u(t,x) \le \delta_1$

Consider now the change of variables

$$\upsilon(t,x) = \left(\frac{u(t,x)}{u(t,x)+\theta}\right)^m$$

then the problem (1.1) can be written as follows:

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{\theta.\upsilon(t,x)^{\frac{1}{m}}}{1-\upsilon(t,x)^{\frac{1}{m}}}\right)^m - \Delta\upsilon(t,x) = 0 \quad , \quad t \in [0,T], \quad x \in \Omega \\ \upsilon(0,x) = \left(\dfrac{\phi(x)}{\phi(x)+\theta}\right)^m \\ \upsilon(t,x) = 0 \quad sur \quad \partial\Omega \end{cases}$$

and $0 = d_0 = \overline{d_0}^m \le \upsilon \le d_1 = \overline{d_1}^m$ (1.3)

we used the implicit temporary discretization

$$\begin{cases} \phi^{(q+1)}\left(\upsilon^{(q+1)}(x)\right) - \delta_t.\Delta\upsilon^{(q+1)}(x) = \phi^{(q)}\left(\upsilon^{(q)}(x)\right) = \overline{\phi}\,,\ dans\,\Omega \\ \upsilon^{(q+1)}(x) = 0 \quad sur\,\partial\Omega,\ q \ge 0 \end{cases} \tag{1.4}$$

we consider the following functional

$$J(\upsilon) = \frac{1}{2}a(\upsilon,\upsilon) - L(\upsilon) + j(\upsilon) \tag{1.5}$$

where

$$a(\upsilon,\upsilon)=\delta_t\int_\Omega \nabla\upsilon\nabla\upsilon dx$$
$$L(\upsilon)=\langle\bar\phi,\upsilon\rangle=\int_\Omega\bar\phi.\upsilon dx$$
$$j(\upsilon)=\sum_{k=1}^{\infty}\frac{\theta}{\left(1+\frac{k}{m}\right)}\upsilon^{1+\frac{k}{m}}$$

the problem (1.4) is equivalent to the following problem

$$\begin{cases} Trouvé\ \upsilon\in K\ tel\ que \\ J(\upsilon)\le J(w) \quad , \quad \forall w\in K \end{cases} \qquad (1.6)$$

where

$$K=\left\{\upsilon\setminus\upsilon\in H_0^1(\Omega), tel\ que\ \ 0=d_0\le\upsilon(x)\le d_1\ sur\ \Omega\right\}$$

and $H_0^1(\Omega)=\left\{\upsilon\in L^2:\frac{\partial\upsilon}{\partial x_i}\in L^2(\Omega), 1\le i\le n, \upsilon|_{\partial\Omega}=0\right\}$

$H_0^1(\Omega)$ normed by

$$\|\upsilon\|_{1,\Omega}=\left(\int_\Omega\left(\|\nabla.\upsilon\|^2+\upsilon^2\right)dx\right)^{\frac{1}{2}}$$

is classically an Hilbert space.

2-Existence and uniquness of the solution

Lemma 1. *If $0 < v < 1$, the mapping $v \longrightarrow j(v)$ is Frechet-differentiable and its derivative is equal to*

$$j'(\upsilon)=\theta\upsilon^{\frac{1}{m}}\sum_{k=0}^{\infty}\upsilon^{\frac{k}{m}}=\frac{\theta\upsilon^{\frac{1}{m}}}{1-\upsilon^{\frac{1}{m}}}$$

Lemma 2. *If $0 = d_0 \le v \le d_1 < 1$, the mapping $v \longrightarrow j(v)$ is convex; moreover, the mapping $v \longrightarrow j'(v)$ is increasing.*

Lemma 3. *The functional $v \longrightarrow g(v)$ is uniformly convex, i.e.*

$$\tau g(\upsilon)+(1-\tau)g(w)-g(\tau\upsilon+(1-\tau)w)\ge c(\Omega)\frac{\tau(1-\tau)}{2}\|\upsilon-w\|_{1,\Omega}^2$$

$\forall\tau\in]0,1[, \forall\upsilon,w\in H_0^1(\Omega)$

where $c(\Omega)$ is a positive constant. Moreover, the mapping $v \longrightarrow g(v)$ is Frechet-differentiable and, respectively, the first and the second Frechet-derivative are equal to

$$\langle g'(v),w\rangle = a(v,w) - L(w) \text{ and } \langle g''(v).w,w\rangle = a(w,w).$$

Corollary 4. *The functional $v \longrightarrow g(v) + j(v) = \frac{1}{2}a(v, v) - L(v) + j(v)$ is uniformly convex.*

Lemma 5. *The functional $v \longrightarrow g(v) = \frac{1}{2}a(v, v) - L(v)$ tends towards to infinity when the norm $\|v\|_{1,\Omega}$ tends to infinity.*

Proposition 6. *Let $J : K \subset H_{1,0} \rightarrow R$ be an uniformly convex function, differentiable on a real Hilbert space $H_{1,0}$, where K is a non-void closed convex set. Then there exists one and only one solution of the constrained optimization problem (1.6).*

3. Multivalued formulation of the problem

The previous model problem equipped with homogeneous Dirichlet boundary conditions, can also be classically formulated like a multivalued problem; indeed, in convex optimization (see [8,17]), the solution of this problem satisfy

$$\begin{cases} Find\ v\in H_{1,0}\ such\ that \\ \bar{A}v-\bar\phi+j'(v)+\partial\psi_K(v)\ni 0\end{cases}$$

where $\partial\psi_K(v)$ is the subdifferential of the indicator function $\psi_K(v)$ of the convex subset K; recall that the indicator function of K defined by

$$\psi_K(v)=\begin{cases}0\ if\ v\in K\\ +\infty\ otherwise\end{cases}$$

and

$$\partial\psi_K(v)=\begin{cases}\emptyset, & if\ v_i<d_0\\ ]-\infty,0], & if\ v_i=d_0\\ 0, & if\ d_0<v_i<d_1\\ [0,+\infty[, & if\ v_i=d_1\\ \emptyset, & if\ v_i>d_1\end{cases}$$

4. Numerical solution of the model problem

4.1.Discretization

In order to simplify the presentation, we will consider that $\Omega\subset\mathbb{R}^d$,$\Omega=[0,1]^d$, with d = 1, 2 or 3. We consider also that Ω is discretized with an uniform mesh $h=\frac{1}{n+1}$, where $n\in\mathbb{N}$, the grid points being constituted by N discretization points, where $N=n^d$ in the case of Dirichlet boundary condition the complete discretization leads, at each time step, to the solution of the following large multivalued nonlinear algebraic system

$$\Phi(V)+\delta_t.AV+\partial\Psi(V)-\bar\phi\ni 0, \quad (4.1)$$

Note that, using such spatial discretizations, the matrix A is irreducibly diagonally dominant; since the diagonal entries of A are positive and all its off-diagonal entries are nonpositive, thus, ***A*** **is a nonsingular M-matrix** (see [13]).

4.2. Parallel subdomain iterative methods without overlapping

Let $\eta\in\mathbb{N}$, be a positive integer and consider now the following block decomposition of problem (4.1) into η subproblems

$$\Phi_i(V^*_i)+\delta_t.A_{i,i}.V^*_i+\delta_t.\sum_{j\ne i}A_{i,j}.V^*_j-\bar\phi_i+\partial\Psi_i(V^*_i)\ni 0,\quad \forall i\in\{1,\dots,\eta\}, \quad (4.2)$$

Then, we associate to the problem (4.2) the following fixed point mapping at it is fixed point V_* if it exists (and we will verify in the sequel that this property is true)

$$V^*_i=(\delta t.A_{i,i})^{-1}(\bar\phi_i-\Phi_i(V^*_i)-\delta t.\sum_{j\ne i}A_{i,j}.V^*_j-w^*_i)=F_i(V^*), (4.3)\quad \forall i\in\{1,\dots,\eta\},$$

Consider now the solution of the subproblems (4.2) by an asynchronous parallel subdomain method without overlapping (see [12,18]) which can be written as follows

$$\begin{cases}\Phi_i(V^{p+1}_i)+\delta t.A_{i,i}.V^{p+1}_i+w^{p+1}_i=\bar\phi_i-\delta t.\sum_{j\ne i}A_{i,j}.W_j, \text{if } i\in s(p)\\ V^{p+1}_i=V^p_i,\ if\ \ i\notin s(p)\end{cases} \quad (4.4)$$

where $W_i^{p+1}=W_i^{p+1}(V^{p+1}_i)$, $\{W_1,\dots,W_{i-1},-,W_{i+1},\dots,W_\eta\}$ are the available values of the components V_j for $j\ne i$, defined by $W_j=V^{\rho_j(p)}{}_j$,

S = $\{s(p)\}_{p\in N}$ is a sequence of non-empty subsets of $\{1, 2, \dots, \eta\}$ that indicates the components updated at the $(p+1)$-th relaxation step and **R** = $\{\rho_1(p),\dots,\rho_\eta(p)\}_{p\in N}$, is a sequence of element of $\mathbb{N}^\eta$; furthermore **S** and **R** verify the following assumptions :

$\forall p\in N$, $s(p)\ne\emptyset$, $\forall i\in\{1,2,\dots,\eta\}$, the set $\{p\in N | i\in s(p)\}$ is infinite,
$\forall i\in\{1,\dots,\eta\}$, $\forall p\in N$, $\rho_j(p)\le p$,
$\forall i\in\{1,\dots,\eta\}$, $\lim_{p\to\infty}\rho_i(p)=+\infty$.

Remark. The algorithm (4.4) describes a computational method where the communications between the processors can be synchronous or asynchronous. For parallel synchronous methods, $\rho(p) = p,\ \forall p \in \mathbb{N}$. *Moreover if* $s(p) = \{1, \ldots, \eta\}$, *i.e.* $\mathbf{S} = \{\{1, \ldots, \eta\}, \ldots \{1, \ldots, \eta\}, \ldots\}$ and $\rho(p) = p,\ \forall p \in \mathbb{N}$, then (4.4) describes the sequential block Jacobi method

If $s(p) = p.mod(\eta) + 1$, *i.e.* $S = \{\{1\}, \{2\}, \ldots, \{\eta\}, \{1\}, \ldots, \{\eta\}, \ldots\}$ and $\rho(p) = p,\ \forall p \in \mathbb{N}$, then (4.4) models the sequential block Gauss–Seidel method. Besides,

If $\boldsymbol{S} = \{\{1\}, \ldots, \{\eta\}, \{\eta\}, \{\eta - 1\}, \ldots, \{1\}, \{1\}, \ldots, \{\eta\}, \{\eta\}, \ldots, \{1\}, \ldots\}$ then (4.4) models the sequential alternating direction method (ADI). This model of parallel asynchronous algorithm therefore appears like a more general model.

4.2.1. Convergence of parallel subdomain methods

For any given vector $W \in R_N$ *let us consider the following implicit fixed point iteration deduced from (4.3)*

$$\Phi i(V_i) + \delta t\,.Aii.V_i + w_i(V_i) = \bar{\phi}_i - \delta t\,.\sum_{j\neq i} Aij.Wj,\quad w_i(V_i) \in \partial\Psi i(Vi\,),$$

$$\forall i \in \{1, \ldots, \eta\},\ (4.5)$$

and then we can set $V = F(W)$. For another vector W' we can write also analogously

$$\Phi i(V'_i) + \delta t\,.Aii.V'_i + w_i(V'_i) = \bar{\phi}_i - \delta t\,.\sum_{j\neq i} Aij.W'j, \forall i \in \{1,\ldots,\eta\}$$

$$where\ w_i(V'_i) \in \partial\Psi i(V'_i\,),\quad (4.6)$$

Subtracting (4.5) and (4.6), we obtain

$$\Phi i(V_i) - \Phi i(V'_i) + Aii.(V_i - V'_i) + w_i(V_i) - w_i(V'_i) = -\sum_{j\neq i} Aij.(Wj - W'j).\ (4.7)$$

Let us denote by $\langle , \rangle_i$ the usual bilinear form associated with a pair of dual spaces, $\|.\|_k$ the classical l_k-norm defined in $\mathbb{R}^{n_i}$ and $\|.\|_k^*$ the norm defined in the dual space; let $g_i \in G_i(V_i - V'_i)$ be an element of the duality map, where $\forall i \in \{1, \ldots, \eta\}$, $\forall k \in [1,\infty]$, g_i satisfies

$$Gi(Vi - V'i) = \{gi \in \mathbb{R}^{n_i} \mid \langle Vi - V'i, gi\rangle_i = \| Vi - V'i \|_k \text{ and } \| gi \|_k^* = 1\};$$

Then by multiplying (4.7) by g_i, we obtain for all i

$$\langle \Phi i(V_i) - \Phi i(V'_i), gi\rangle_i + \langle w_i(V_i) - w_i(V'_i), gi\rangle_i + \delta t.\langle Aii.(V_i - V'_i), gi\rangle i$$

$$= -\delta t\,.\sum_{j\neq i} \langle Aij.(Wj - W'j), gi\rangle i$$

A being an M-matrix, the diagonal submatrices A_{ii}, $\forall i \in \{1, \ldots, \eta\}$, are also **M**-matrices. Applying a characterization of **M**-matrices from [19], these submatrices are strongly accretive matrices, i.e. $\forall i \in \{1, \ldots, \eta\}$ the following inequality holds

$$\langle Aii.(V_i - V'_i), gi\rangle i \geq \mu_{ii} \| Vi - V'i \|_k,\qquad (\mu_{ii} > 0);\quad (4.8)$$

since the subdifferential mapping is maximal monotone and the operator Φ_i is increasing and consequently also maximal monotone, then the left hand side of the previous relation can be minored by $\mu_{ii} \| Vi - V'i \|_k$ Concerning the right hand side of the previous relation, the mapping $\langle ., .\rangle_i$ being a bilinear form, $\forall j \in \{1, \ldots, \eta\}, j \neq i$, for all $k \in [1,\infty]$, we obtain the following upper bound

$$\sum_{j\neq i} \langle Aij.(Wj - W'j), gi\rangle i \leq \sum_{j\neq i} \mu_{ij} \| Wj - W'j \|_k,\quad \mu_{ij} > 0);\quad (4.9)$$

where μ_{ij} denotes the subordinate matrix norm of A_{ij} associated with the scalar norm $\|.\|_k$. Taking into account relations (4.8) and (4.9) we obtain the following inequality

$$\| Vi - V'i \|_k \leq \sum_{j\neq i} \frac{\mu_{ij}}{\mu_{ii}} \| Wj - W'j \|_k,\quad \forall i \in \{1,\ldots,\eta\}\ (4.10)$$

Let us now denote by $\bar{\mathcal{g}}$ the following $(\eta \times \eta)$ matrix

$$\bar{\mathcal{g}}_{ij} = \begin{cases} 0\ if\ i = j \\ \frac{\mu_{ij}}{\mu_{ii}}\ if\ i \neq j \end{cases}$$

$\bar{\mathcal{g}}$ is a nonnegative matrix. Moreover let us define the vectorial norm of a vector Y, by the positive vector of $\mathbb{R}^{\eta}$, the components of which are

$$\mathcal{y} \to |\mathcal{y}| = \{\ldots, \|Y_j\|_k, \ldots\};$$

thus, the inequalities (4.10) can be written as follows

$$|V_i - V'_i| \leq \bar{\mathcal{g}}.|Wj - W'j|,\qquad \forall W,\qquad (4.11)$$

Note that the matrix $\bar{\mathcal{g}}$ with diagonal entries null and off-diagonal entries equal to $\frac{\mu_{ij}}{\mu_{ii}}$ is the Jacobi matrix of a matrix $\mathcal{M}$ with diagonal entries equal to μ_{ii} and off-diagonal entries equal to $-\mu_{ij}$. If $\mathcal{M}$ is an irreducible **M**-matrix then $\bar{\mathcal{g}}$ is an irreducible and non-negative matrix and all eigenvalues of $\bar{\mathcal{g}}$ have a modulus less than one. Let us denote by ν the spectral radius of $\bar{\mathcal{g}}$ and by $\Gamma \in \mathbb{R}^{\eta}$ the associated eigenvector. Classically by the **Perron–Frobenius** theorem, all the components of Γ are strictly positive and the following inequality $\bar{\mathcal{g}}\Gamma \leq \nu\Gamma$ is valid, where $0 \leq \nu < 1$ (see [20]). If we consider the weighted uniform norm defined by

$$\|V\|_{\Gamma,\infty} = \max_{1\leq i\leq\eta}\left(\frac{\|V_i\|_k}{\Gamma_i}\right)$$

then, by a straightforward way, we obtain

$$\|F(V) - F(V')\|_{\nu,\Gamma} \leq \nu\|V - V'\|_{\nu,\Gamma},\qquad \forall\, V, V' \in \mathbb{R}^N,\qquad (4.12)$$

Then, using the last inequality (4.12), F is a contraction and we obtain a result of existence and uniqueness of both the fixed point of F and of the solution of Problem (4.1). Then, if the previous assumptions are verified, i.e. $\mathcal{M}$ is an irreducible **M**-matrix, whatever be the initial guess V_0, the convergence of the parallel asynchronous, synchronous and sequential iterations described by (4.4), results now either from [12,18] associated to the property of contraction with respect of a vectorial norm (4.11) of the mapping F or directly of the contraction in the usual sense (4.12) by applying the result of [21]. Then we can formulate the following result.

Proposition 1

$\partial\Psi$ being a diagonal increasing operator, under the following assumptions :

- the matrix $\mathcal{M}$ is an M-matrice.
- Φ a diagonal increasing operator.

the parallel asynchronous, synchronous and sequential iterative subdomain method without overlapping defined by (4.4) converge to V^*.

Remark1

Practically, the algebraic system to solve is split into $\bar{\eta}$ blocks, $\bar{\eta} \leq \eta$, contiguous blocks, corresponding to a coarser subdomain decomposition without overlapping

proposition 2

Consider the solution of the algebraic system (4.1) by the parallel asynchronous relaxation methods (4.4), under the assumptions of Proposition 1. Then, the sequential and the parallel synchronous and asynchronous subdomain methods without overlapping (4.4) converge to the solution of the problem (4.1) for every coarser subdomain decomposition.

Proof

Indeed, considering a point decomposition of the model problem, we can state a similar result than the one stated in Proposition 1 for this particular decomposition and applying a result of [15], the proof follows from a straightforward way for every coarser subdomain decomposition.

5-Numerical experiments

the iterative scheme that computes $V_{(q+1)}$ is the Newton algorithm:

$$\begin{cases} V^{q+1,0} = V^q \\ (\delta_t A + \Phi'(V^{q+1,p})\delta V = -(\Phi(V^{q+1,p}) + \delta_t A V^{q+1,p} - \Phi(V^q) \\ \bar{V}^{q+1,p+1} = V^{q+1,p} + \delta V, \\ V^{q+1,p+1} = proj(\bar{V}^{q+1,p+1}) \end{cases}$$

Where

$$\Phi'(z) = \frac{\theta . z^{\frac{1}{m}-1}}{m(1 - z^{\frac{1}{m}})^2}$$

d=1		d=2		d=3	
Time steps	Discr.points	Time steps	Discr.points	Time steps	Discr.points
20	1000	20	50*50	20	50*50*50

Table1: Number of time steps and number of discretization points on each axis for d = 1, 2, 3.

Newton-relaxation 3D			Newton-relaxation 2D			Newton-Gauss 1D	
Time	Linear	G.S.iterat	Time	Linear	G.S.iterat	Time	Linear
1458.9	2	60 (1)	9.42	2	16 (1)	5.03	4 or 3
-	-	59 (2)	-	-	1 (2)	-	-

Table 2: Elapsed time (seconds), number of linearizations and average number of Gauss–Seidel iterations for each linearization phase for d = 2, 3.

6. Conclusion

In the present study we have solved the modified porous medium equation by a numerical way; such solution has been possible when Dirichlet boundary conditions are considered. Due to the constraints on the solution, the more convenient formulation of the problem is obtained by perturbation to the problem by the subdifferential mapping of the indicator function of the convex set describing these constraints. After appropriate assumptions, particularly the facts that the spatial discretization matrix is an M-matrix and also that the affine system is perturbed by increasing diagonal operators, this formulation allows to study in a unified way the behavior of the sequential and parallel relaxation methods used for the solution of the discretized problem by various subdomains methods. Parallel experiments show the efficiency of the studied method.

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